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NON-EUCLIDEAN GEOMETRY: HISTORICAL AND EXPOSITORY.

By **GEORGE BRUCE HALSTED, A. M., (Princeton), Ph. D., (Johns Hopkins), Member of the London Mathematical Society, and Professor of Mathematics in the University of Texas, Austin, Texas.**

[Continued from the December Number.]

SCHOLION I. Here it is permitted to observe a notable difference from the hypothesis of acute angle.

For in this the general concurrence of straights cannot be demonstrated in this way, as often as any straight falling upon two, makes two internal angles toward the same parts less than two right angles; cannot, I say, be directly demonstrated, even if in this hypothesis the aforesaid general concurrence be admitted, as often as one of the two angles is right.

For although the straight AD be perpendicular even to the straight AP; in which case it certainly could not concur with another perpendicular PL (Eu. I. 17.); nevertheless the two angles together DAX, PXA, could be less than two right angles, in accordance with the aforesaid hypothesis, since in it the two angles together PAX, PXA may be less (P. IX.) than one right angle.

But it was worth while to have observed this. Just as, infact, solely from the admission of this general concurrence when one of the angles is right, and with an assigned incident however small, the hypothesis of acute angle can be demolished; this we will show after the three next propositions.

[To be continued.]

ARITHMETIC.

Conducted by B. F. FINKEL, Kidder, Mo. All contributions to this department should be sent to him.

SOLUTIONS TO PROBLEMS.

33. Proposed by F. P. MATZ, M. Sc., Ph. D., Professor of Mathematics and Astronomy in New Windsor College, New Windsor, Maryland.

A wine-merchant's *apparent* profit is 25% of his sales which are 10% of cost-less water. What is his actual rate per cent. of profit?

I. Solution by P. S. BERG, Apple Creek, Ohio, and the PROPOSER.

The *apparent* selling-price is $\frac{125}{100}$ of the cost. On account of the *costless* water, the *actual* selling-price is ($\frac{10}{9}$ of $\frac{125}{100}$) of 100% $=138\frac{8}{9}$%. Hence, the actual rate per cent. of profit is $38\frac{8}{9}$%.

II. Solution by FRANK HORN, Columbia, Missouri, and Professor H. J. GAERTNER, Wilmington College, Wilmington, Ohio.

II.
1. 100% = apparent value.
2. 125% = selling price of apparent value.
3. 90% = 100% − 10% = value of quantity sold for 125%.
4. ∴ $\frac{125}{90}$% = what 1% sells for.
5. $138\frac{8}{9}$% $=100\times\frac{125}{90}$% = real selling price.
6. 100% = true value.
7. $38\frac{8}{9}$% $=138\frac{8}{9}$% − 100% = rate of gain.

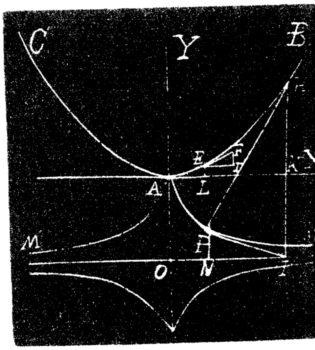
∴ The actual rate of gain is $38\frac{8}{9}$%.

34. A chain 100m long, weighing 14 oz. to the foot, is suspended from points on a level 80m apart. What is the sag, the batter at the ends, and the horizontal tension? [From *Wentworth & Hill's High School Arithmetic.*]

Solution by B. F. FINKEL, A. M., Professor of Mathematics and Physics in Kidder Institute, Kidder, Missouri.

The form of the chain fulfilling the conditions of the problem is the curve known as the *catenary*. Let B and C be the points of suspension of the chain, E any point in the chain, $AL=x$, $EL=y$.

Let $AE=s$ and $w=45.93$oz. the weight of a metre of length of the chain.



Then ws = the weight of the portion AE = the load suspended at E, or the vertical tension at E. Let aw = the horizontal tension at A, the weight of a units of length. Let EF be a tangent at E; then if EF represents the tension at E, EI and IF will represent the horizontal and vertical tensions respectively, at E.

Hence, $\frac{dy}{dx}=\frac{FI}{EI}=\frac{ws}{wa}=\frac{s}{a}\dots\dots(1)$. But $ds=\sqrt{(dy^2+dx^2)}$. ∴ $dy=\sqrt{(ds^2-dx^2)}$.

$s/a=\sqrt{(ds^2-dx^2)}/dx$, whence

$$\frac{dx}{ds}=\frac{a}{\sqrt{(a^2+s^2)}}. \quad \therefore \quad x=a\int\frac{ds}{\sqrt{(a^2+s^2)}}$$

$=a\log_e(s+\sqrt{a^2+s^2})+c$. Since $x=0$, when $s=0$, $c=-a\log a$.

∴ $x=a\log_e[(s+\sqrt{a^2+s^2})/a]\dots\dots(2)$. From (2), we have

$s=\frac{a}{2}(e^{\frac{x}{a}}-e^{-\frac{x}{a}})\dots\dots(3)$. From (1) and (3) $\frac{dy}{dx}=\frac{1}{2}(e^{\frac{x}{a}}-e^{-\frac{x}{a}})$.

∴ $y=\frac{a}{2}(e^{\frac{x}{a}}-e^{-\frac{x}{a}})+c$. Since $y=0$ when $x=0$, $c=-a$.

∴ $y=\frac{a}{2}(e^{\frac{x}{a}}-e^{-\frac{x}{a}})-a\dots\dots(4)$. From (3) and (4) we get $a=(s^2-y^2)/2y$.